The $M_\bullet$ - T relation for a black hole in thermodynamic equilibrium with the surrounding intergalactic medium

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ABSTRACT
I consider a toy model of self-regulated black hole accretion. The black hole grows through Bondi accretion and a fraction of the accretion power is distributed as thermal feedback into the surrounding gas. The gas expands or contracts until AGN heating and radiative cooling balance each other. The balance of heating and cooling is used to determine a quasi-equilibrium temperature at which the black hole accretes in self-regulated equilibrium with the surrounding intergalactic medium. This temperature grows with the black hole mass. The temperature increase is very steep around a critical black hole mass due to the shape of the cooling function. The quasi-equilibrium temperature cannot exceed the virial temperature or the AGN will drive a thermal wind. This limits the black hole mass to a maximum value determined by the depth of the potential well. In the regime in which cooling is dominated by bremsstrahlung, this model determines a relation between black hole mass and halo characteristic velocity of the form $M_\bullet \propto v^4$. The predictions of the model are consistent with the observed black hole mass – bulge velocity dispersion relation.

Key words: galaxies: formation, active, ISM – quasars: general – black hole physics

1 INTRODUCTION

All massive spheroids for which the data are sufficiently good show evidence for a central black hole. The relation between black hole mass and bulge mass (Marconi & Hunt 2003; Häring & Rix 2004) suggests that the black hole contains $\sim 0.2\%$ of the bulge stellar mass. The gravitational binding energy released by matter accreted onto the black hole is $\sim 10\%$ of the rest mass energy. A small fraction of this energy is sufficient to affect the dynamics and the thermal state of the intergalactic medium (IGM) dramatically.

Active galactic nucleus (AGN) heating was introduced to solve the cooling flow problem in galaxy clusters (Tabor & Binney 1993; Binney & Tabor 1995; Tucker & David 1997; Ciotti & Ostriker 1997; Cavaliere et al. 2002; Ruszkowski & Begelman 2002) and the related problem of the entropy floor (Roychowdhury et al. 2004; Lapi et al. 2005). Silk & Rees (1998) went further and proposed that AGN outflows are an essential part of the galaxy formation process. In this scenario, it is AGN feedback that determines the observed relation between black hole mass and velocity dispersion of the host bulge (Ferrarese & Merritt 2000; Gebhardt et al. 2000; Merritt & Ferrarese 2001; Tremaine et al. 2002). King (2003), Granato et al. (2004), Murray et al. (2005)

and Di Matteo et al. (2005) have developed more elaborate versions of the original idea by Silk & Rees (1998) that AGN winds terminate star formation in the host galaxy (the reference list does not pretend to be complete).

The interaction of the black hole with the IGM is a complex problem, which requires hydrodynamic simulations (e.g. Quilis et al. 2001; Reynolds et al. 2001; Churazov et al. 2002; Reynolds et al. 2002; Basson & Alexander 2003; Omma & Binney 2004; Ruszkowski et al. 2004; Brüggen et al. 2005; Di Matteo et al. 2005; Zanni et al. 2005), but here I want to concentrate on a simple aspect, where an analytic approach is possible and can offer a better insight into the results of hydrodynamic simulations.

I consider a black hole at the centre of a profile with a core, where the gas is approximately homogeneous and isothermal. The black hole accrete mass and returns heat to its surrounding environment. The core expands or contracts until AGN heating and radiative cooling balance each other. The balance of heating and cooling determines the relation between the mass of the black hole and the temperature of the IGM at the equilibrium. If the equilibrium temperature is much larger than the virial temperature, the system will only admit outflow solutions.

2   MODEL OF THE BH – IGM COUPLING

A black hole of mass $M_\bullet$ surrounded by gas with density ρ and speed of sound c_s accretes at a rate determined by the Bondi (1952) formula:

$$\dot{M}_\bullet = 4\pi\alpha(GM_\bullet)^2 \frac{\rho}{c_s^3} = 4\pi\alpha(GM_\bullet)^2 \rho \left(\gamma\frac{kT}{\mu m_p}\right)^{-\frac{3}{2}}, \quad (1)$$

where G is the gravitational constant and α is a number of order unity. In the second equality, the speed of sound c_s is rewritten in terms of the temperature T. Here γ is the adiabatic index, k is the Boltzmann constant, m_p is the proton mass and μ is the mean particle mass as a fraction of the proton mass. Eq. (1) assumes that the accretion is spherical and that the gas is approximately static at infinity, but is widely used to estimate $\dot{M}_\bullet$ even when these conditions are not satisfied (e.g. Di Matteo et al. 2005).

A fraction β of the power generated by the accretion of matter onto the black hole heats the surrounding gas at a rate $\dot{Q}_{\rm heat}$. If the heat is distributed over a gas mass $M_{\rm gas}$, then the heating rate per unit mass is:

$$\frac{\dot{Q}_{\rm heat}}{M_{\rm gas}} = \frac{\beta\epsilon\dot{M}_\bullet c^2}{M_{\rm gas}} = 4\pi\alpha\beta\epsilon c^2 \,(GM_\bullet)^2 \frac{\rho}{M_{\rm gas}}\left(\gamma\frac{kT}{\mu m_p}\right)^{-\frac{3}{2}} =$$

$$= 630\,\alpha\beta\,\epsilon_{0.1}\gamma_{\frac{5}{3}}\mu_{\frac{16}{27}}X_{\frac{3}{4}}^{-1}\,\frac{M_{\bullet 8}^2}{M_{\rm gas\,8}}\,n_H T_6^{-3/2}\,{\rm erg\,s^{-1}cm^3 g^{-1}}, (2)$$

where c is the speed of light and $\epsilon \sim 0.1$ is the energetic efficiency of black hole accretion. In the second line, where we insert the numerical values, $\epsilon_{0.1} \equiv \epsilon/0.1$, $\gamma_{5/3} \equiv 3\gamma/5$, $\mu_{16/27} \equiv 27\mu/16$, $X_{3/4} \equiv 4X_p/3$ (X_p is the hydrogen baryon fraction), $M_{\bullet 8} \equiv M_\bullet/10^8 M_\odot$, $M_{\rm gas\,8} \equiv M_{\rm gas}/10^8 M_\odot$ and $T_6 \equiv T/10^6{\rm K}$, while n_H is the number density of hydrogen atoms (free protons for an ionised gas).

The cooling rate per unit mass of the heated gas is:

$$\frac{\dot{Q}_{\rm cool}}{M_{\rm gas}} = \frac{1}{\rho}\Lambda(T,Z)\,n_H^2 = \frac{X_p}{m_p}\Lambda(T,Z)\,n_H = \qquad (3)$$

$$= 45\,X_{\frac{3}{4}}\,\Lambda_{-22}(T,Z)\,n_H\,{\rm erg\,s^{-1}cm^3 g^{-1}},$$

where Λn_H^2 is the radiated power per unit volume. Λ is a function of the temperature T and the metallicity Z, and $\Lambda_{-22} \equiv \Lambda/10^{-22}\,{\rm erg\,s^{-1}cm^3}$. In the next Section I proceed to examine under what conditions the heating term and the cooling term balance each other off.

3   LOCAL $M_\bullet$ – T EQUILIBRIUM RELATION

Both the heating (Eq. 2) and the cooling (Eq. 3) rate scale linearly with the density of the gas. The difference is in the temperature scaling. Fig. 1 compares the the rates of heating and cooling as a function of temperature. The curves are computed from the collisional ionisation equilibrium cooling function of Sutherland & Dopita (1993) and show $\dot{Q}_{\rm cool}(T)/(n_H M_{\rm gas})$ for different metallicities. The diagonal straight lines show the heating term $\dot{Q}_{\rm heat}(T)/(n_H M_{\rm gas})$ and are parameterised by the black hole – IGM coupling factor $\zeta = \alpha\beta\delta M_{\bullet 8}^2/M_{\rm gas\,8}$ (Eq. 2), where δ is the ratio between the gas density at the Bondi radius (in Eq. 2) and the gas density at the radius that contains a gas mass $M_{\rm gas}$ (in Eq. 3).

At very low T, the heating term always dominates and
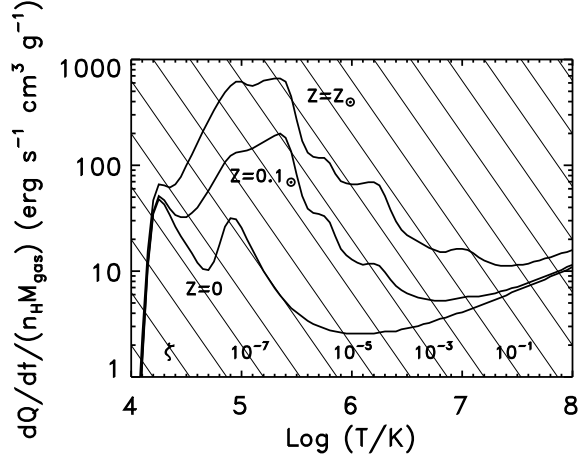
Figure 1. $\dot{Q}_{\rm heat}(T,\zeta)/(n_H M_{\rm gas})$ (diagonal lines) and $\dot{Q}_{\rm cool}(T,Z)/(n_H M_{\rm gas})$ (curves) for different values of the black hole – IGM coupling constant ζ and the metallicity Z.

the temperature increases. At very high T, the cooling term dominates and the temperature goes down. In both cases, T will converge to an equilibrium temperature $T_{\rm eq}$ determined by two parameters: Z and ζ.

For each metallicity, there is a value of ζ, which we call $\zeta(Z)$, where the equilbrium extends over a temperature range (Fig. 1). For zero metals, this value is $\zeta(Z) \simeq 10^{-3}$ and the temperature range is $10^5\,{\rm K} \lesssim T_{\rm eq} \lesssim 3\times10^5\,{\rm K}$. For $Z = 0.1\,Z_\odot$, $\zeta(Z) \simeq 0.025$ and the temperature range is $2\times10^5\,{\rm K} \lesssim T_{\rm eq} \lesssim 10^6\,{\rm K}$. For Solar abundances, $\zeta(Z) \sim 0.1$ and the temperature range is $2\times10^5\,{\rm K} \lesssim T_{\rm eq} \lesssim 3\times10^6\,{\rm K}$. If the parameter ζ, which measures the effective efficiency of AGN heating, is $\zeta < \zeta(Z)$, then cooling dominates at all but the lowest temperatures and AGN feedback is negligible.

For $\zeta \gg \zeta(Z)$, cooling is dominated by bremsstrahlung. In this regime, the cooling function can be approximated as $\Lambda_{-22} \sim 0.2\sqrt{T_6}$, so that equating (2) and (3) gives:

$$T_{\rm eq} \sim 8\times10^6\,(\alpha\beta\delta\,\epsilon_{0.1}\gamma_{\frac{5}{3}}\mu_{\frac{16}{27}})^{\frac{1}{2}}X_{\frac{3}{4}}^{-1}\,\frac{M_{\bullet 8}}{M_{\rm gas\,8}^{\frac{1}{2}}}\,{\rm K}. \qquad (4)$$

The equilibrium temperature is proportional to the black hole mass. The constant of proportionality depends on the black hole accretion efficiency with respect to the Bondi rate, on the energetic efficiency of AGN heating and on the gas mass on which the feedback is distributed.

The equilibrium described in this Section is a local quasi-equilibrium. It is a quasi-equilibrium because $T_{\rm eq}$ is not constant with time. The black hole accretes at the rate specified by Eq. (1) and $T_{\rm eq}$ must constantly increase to keep up with the increasing black hole mass ($\zeta \propto M_\bullet^2$). However, substituting Eq. (4) into Eq. (1) shows that the quasi-equilibrium accretion rate is $\dot{M}_\bullet \propto M_\bullet^{1/2}$, while the accretion rate without self-regulation would be $\dot{M}_\bullet \propto M_\bullet^2$ until the Eddington limit is reached. This model predicts that in the self-regulated regime $\dot{M}_\bullet/\dot{M}_{\rm Eddington} \propto M_\bullet^{-1/2}$. The equilibrium is local because we made no assumption for the scale on which the feedback is distributed, which may be as small as the nuclear region or as large as a galaxy cluster. In the next Section we shall see how relating the feedback scale to the properties of the host system can generate a limit mass for black hole growth.

4 THE $M_\bullet - T_{\rm VIR}$ EQUILIBRIUM RELATION IN GALAXY CLUSTERS

If AGN heating is relevant not only for the self-regulation of the black hole and for the gas in the central $\sim 1\,{\rm kpc}$, but also to keep the cluster gas hot, then the feedback must be distributed on a scale of the order of the radius of the cluster core ($\sim 100\,{\rm kpc}$). In this scenario, one assumes that $T_{\rm eq} \sim T_{\rm vir}$ and that $M_{\rm gas}$ is equal to the mass of the gas in the core of the hot gas distribution, and one uses the equation $\dot{Q}_{\rm heat}(T)/(n_{\rm H}M_{\rm gas}) = \dot{Q}_{\rm cool}(T)/(n_{\rm H}M_{\rm gas})$ to determine the equilibrium black hole mass.

Komatsu & Seljak (2001) have assumed that the dark matter distribution in a virialised halo is described by the Navarro et al. (1997) profile and have used this profile to compute analytic hydrostatic equilibrium solutions for the baryonic component (I verified with computer simulations that these are truly hydrostatic solutions). They contain three parameters: the virial mass $M_{\rm vir}$, the virial radius $r_{\rm vir}$ and the mass fraction in hot gas, which, for a cluster, is of the order of the cosmic baryon fraction (~ 0.1). The first two are not independent when a critical density contrast (a redshift of collapse) is specified. The relation between the mass of the black hole $M_\bullet$ and the virial velocity $v_{\rm vir} = {\rm G}M_{\rm vir}/r_{\rm vir}$ is determined from Eq. (4), where $T_{\rm eq} \sim {\rm G}\mu m_{\rm p}M_{\rm vir}/(3kr_{\rm vir})$ and $M_{\rm gas}$ is determined by the Komatsu & Seljak (2001) model.

In an isothermal sphere the virial velocity $v_{\rm vir}$ and the velocity dispersion σ are related by $v_{\rm vir} = \sigma\sqrt{2}$. Fig. 2 shows the $M_\bullet - \sigma$ relation between the mass of the black and the velocity dispersion of the host galaxy if the stars in the bulge of the central galaxy have the same velocity dispersion as the dark matter. The two lines correspond to different redshifts of collapse of the dark matter halo. The redshift of collapse $z_{\rm c}$ is used to calculate the virial overdensity with the fitting formulae of Bryan & Norman (1998) and thus to compute $r_{\rm vir}$ as a function of $M_{\rm vir}$. The data points show the observed black hole mass – bulge velocity dispersion relation.

The model and the data overlap already in the simple case $\alpha \sim \beta \sim \delta \sim 1$ (Fig. 2). One must exert great caution in relating the virial velocity of the cluster $v_{\rm vir}$ to the velocity dispersion of the central galaxy σ. A more careful analysis would show that $v_{\rm vir}/\sqrt{2}$ is an overestimate of the velocity dispersion of the central galaxy, which is compensated by an underestimate of δ, since the gas density is higher at the centre due to the presence of the central galaxy.

I will illustrate this point by considering the special case of the radio source in M87, at the centre of the Virgo cluster. If we assume that the Virgo cluster has a mass of $M_{\rm vir} \sim 1.5 \times 10^{14}\,M_\odot$ and has virialised at low redshift, then $v_{\rm vir} \sim 690\,{\rm km\,s}^{-1}$ and $T_{\rm vir} \sim 10^7\,{\rm K}$. If we also assume that the baryons are in the form of a hot gas described by the model of Komatsu & Seljak (2001), then the predicted values for the central electron density and the mass of the baryons in the cluster core are $n_{\rm e} \sim 0.004\,{\rm cm}^{-3}$ and $M_{\rm c} \sim 1.8 \times 10^{12}\,M_\odot$ (in reality they will both be lower because some of the baryons are in stars). The virial temperature $T_{\rm vir}$ is consistent with the X-ray temperature in the core of the Virgo cluster (e.g. Di Matteo et al. 2003), but $v_{\rm vir}/\sqrt{2} \sim 500\,{\rm km\,s}^{-1}$ is much larger than the $\sigma \sim 375\,{\rm km\,s}^{-1}$ determination by Tremaine et al. (2002) for the stellar velocity dispersion in M87. Harms et al. (1994) and Macchetto et al. (1997) have estimated that M87 con-
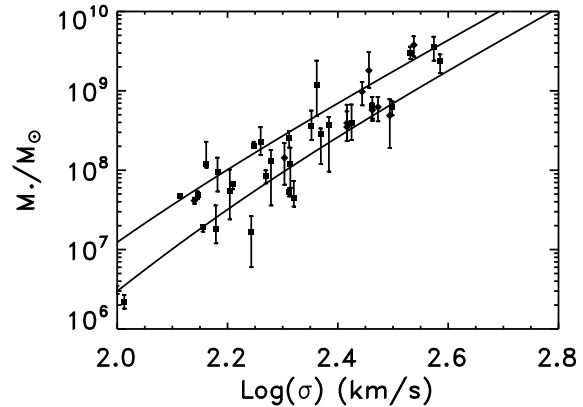
Figure 2. The predicted $M_\bullet - v_{\rm vir}/\sqrt{2}$ relation (lines) compared with the observed $M_\bullet - \sigma$ relation (points with error bars), where σ is the velocity dispersion of the host bulge. The model is for bremsstrahlung cooling (Eq. 4). The upper line corresponds to a redshift of collapse of $z_{\rm c} \simeq 2$. The lower line corresponds to $z_{\rm c} \simeq 0$ and thus to a lower halo density. The squares with error bars are the black hole mass estimates of Tremaine et al. (2002). The diamonds with error bars are those of Ferrarese & Merritt (2000).

tains a supermassive black hole of $M_\bullet \sim 3 \times 10^9\,M_\odot$. Di Matteo et al. (2003) have used this estimate for the black hole mass together with their X-ray observations of M87 and have shown that the mechanical power of the jet in M87 can be understood if the black hole accretes the surrounding hot gas at the Bondi rate ($\alpha \sim 1$) and most of the accretion power is released mechanically ($\beta \sim 1$). They have also observed that the central density of the gas in M87 is higher than our estimate from the Komatsu & Seljak (2001) model by a factor of $\delta \sim 30 - 40$. With these values, Eq. (4) gives $M_{\rm gas} \sim 2 \times 10^{12}\,M_\odot$, which is indeed consistent with our estimate for the mass of the baryons in the core of the Virgo cluster.

With this normalisation issue in mind, the comparison in Fig. 2 demonstrates that this model can reproduce the type of relation observed in the data. The predicted $M_\bullet - \sigma$ relation is not exactly a power law, but is consistent with a power law relation of the form $M_\bullet \propto \sigma^\eta$ with $\eta \sim 4$.

5 DISCUSSION AND CONCLUSION

I have considered a very simple model of self-regulated black hole accretion. The black hole grows through spherical Bondi accretion and a fraction of the accretion power is distributed as thermal feedback into the surrounding IGM. The heating rate per unit mass of the gas depends on three parameters: the accretion efficiency with respect to the Bondi rate, α, the fraction of the accretion power converted into heat, β, and the mass $M_{\rm gas}$ on which the heat is distributed. If $M_{\rm gas}$ is much larger than the mass within the Bondi radius and the interaction between the AGN and the IGM happens at large scales (e.g. $\sim 100\,{\rm kpc}$), then one must also deal with the complication that the gas affected by feedback has a different density from the gas that feeds the black hole.

If the black hole is not producing enough heat or if the

heat is distributed on a very large scale, the thermal energy input is radiated almost immediately and the equilibrium temperature is not much higher than the IGM temperature without AGN feedback. For a given metallicity, there is a critical heating rate per unit mass at which the heating and the cooling rates are comparable over a broad temperature range. This is the range where the cooling function decreases with the temperature. Above this critical rate, the gas is only able to cool at high temperatures, in the bremsstrahlung regime. For a given set of parameters, the critical heating rate corresponds to a critical black hole mass. Feedback is inefficient below this critical black mass, while it can rapidly heat the gas to very large temperatures after the black hole has passed this threshold.

In the Bondi model without self-regulation, the black hole grows with $\sim M_\bullet^2$. Self-regulation reduces the power with which the accretion rate depends on the black hole mass ($\sim M_\bullet^{0.5}$ for cooling by bremsstrahlung).

If the equilibrium temperature is higher than the virial temperature, the black hole drives a thermal wind and the approximation that the heating is quasi-static breaks down. This blow-out condition implies a maximum $M_\bullet(T_{\rm vir})$, which gives rise to a relation of the type $M_\bullet \propto v_{\rm vir}^4$.

The question is the relevance of the approximations made by the model:

i) Bondi spherical accretion in a homogeneous medium supported by thermal pressure may be a bad estimate of how $\dot{M}_\bullet$ depends on the black hole environment, particularly if the gas that feeds the black hole is cold and clumpy.

ii) In a multiphase IGM, the phase which feeds the black hole may not be the same that receives the heat. One can imagine a scenario in which the AGN is fuelled through the accretion of cold clouds, but most of the AGN heating goes to the dilute hot gas that fills the space between the clouds. This is the most likely picture in powerful AGN triggered by galaxy mergers.

iii) Feedback may be mechanical and only thermalised on very large scales after escaping from the galactic core in a collimated outflow. In our model this corresponds to a large value of $M_{\rm gas}$, but the difference between the accretion scale and the thermalisation scale may determine an oscillation pattern rather than an equilibrium solution (e.g. Omma & Binney 2004).

iv) The onset of a thermally driven wind invalidates the assumption that heating is quasi-static. In the presence of outflow rather than static boundary conditions, the Bondi formula greatly overestimates the black hole accretion rate (e.g. Di Matteo et al. 2003). This will almost certainly be the case if the quasi-equilibrium temperature is much higher than the virial temperature of the system.

The toy model presented in this paper is mostly relevant to low power AGNs fed through the accretion of hot gas in the core of massive haloes, but it provides a general rule of thumb to determine under what conditions AGN feedback is important and self-regulation is possible.

6   ACKNOWLEDGEMENTS

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